Feedback linearization with a neural network based compensation scheme

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Abstract. This paper presents a nonlinear controller for uncertain single-input–single-output (SISO) nonlinear systems. The adopted approach is based on the feedback linearization strategy and enhanced by a Radial Basis Function neural network to cope with modeling inaccuracies and external disturbances that can arise. An application of this nonlinear controller to an electro-hydraulic actuated system subject to an unknown dead-zone input is also presented. The obtained numerical results demonstrate the improved control system performance.

Keywords: Electro-Hydraulic Systems, Feedback Linearization, Neural Networks, Nonlinear Control, Radial Basis Functions.

1 Introduction

Due to its simplicity, feedback linearization scheme is commonly applied in industrial control systems, specially in the field of industrial robotics. The main idea behind this control method is the development of control law that allows the transformation of the original dynamical system into an equivalent but simpler one. Although feedback linearization represents a very simple approach, an important drawback is the requirement of a perfectly known dynamical system, in order to ensure the exponential convergence.

Due to the adaptive capabilities of the artificial neural networks, it has been largely employed in the last decades to both control and identification of dynamical systems. In spite of the simplicity of this heuristic approach, in some situations a more rigorous mathematical treatment of the problem is required. Recently, much effort has been made to combine artificial neural networks with nonlinear control methodology [2–4, 6]

In this paper, a nonlinear controller is proposed to deal with uncertain single-input-single-output (SISO) nonlinear systems. The adopted approach is based on the feedback linearization method, but enhanced by a neural network compensation scheme to cope with modeling inaccuracies and external disturbances.

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Radial basis functions are used as activation functions and the related tracking error as input. Numerical simulations are carried out in order to demonstrate the improved performance of the proposed control scheme.

2 Control scheme

Consider a class of n^{th}-order nonlinear systems:

$$x^{(n)} = f(\mathbf{x}, t) + b(\mathbf{x}, t)u + d \tag{1}$$

where u is the control input, the scalar variable x is the output of interest, $x^{(n)}$ is the n-th time derivative of x, $\mathbf{x} = [x, \dot{x}, \ldots, x^{(n-1)}]$ is the system state vector, d represents external disturbances and unmodeled dynamics, and $f, b : \mathbb{R}^n \to \mathbb{R}$ are both nonlinear functions.

Let us now define an appropriate control law that ensures the tracking of a desired trajectory $\mathbf{x}_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]$, *i.e.* the controller should assure that $\tilde{\mathbf{x}} \to 0$ as $t \to \infty$, where $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d = [\tilde{x}, \dot{\tilde{x}}, \ldots, \tilde{x}^{(n-1)}]$ is the related tracking error. On this basis, assuming that the state vector $\mathbf{x}$ is available to be measured and the functions f and b are well known, with $|b(\mathbf{x}, t)| > 0$, the following control law:

$$u = b^{-1}(-f + x_d^{(n)} - k_0\tilde{x} - k_1\dot{\tilde{x}} - \cdots - k_{n-1}\tilde{x}^{(n-1)} - d) \tag{2}$$

guarantees that $\mathbf{x} \to \mathbf{x}_d$ as $t \to \infty$, if the coefficients k_i $(i = 0, 2, \ldots, n-1)$ make the polynomial $p^n + k_{n-1}p^{n-1} + \cdots + k_0$ a Hurwitz polynomial.

The convergence of the closed-loop system could be easily established by substituting the control law, Eq. (2), in the nonlinear system, Eq. (1). The resulting dynamical system could be rewritten by means of the tracking error:

$$\tilde{x}^{(n)} + k_{n-1}\tilde{x}^{(n-1)} + \ldots + k_1\dot{\tilde{x}} + k_0\tilde{x} = 0 \tag{3}$$

where the related characteristic polynomial is Hurwitz.

However, since d is unknown the control law in Eq. (2) is not sufficient to ensure the exponential convergence of the tracking error to zero. On this basis, we propose the adoption of a neural network within the control law, in order to estimate d and to enhance the feedback linearization performance. Considering $\hat{d}$ the output of the neural network, the control law becomes:

$$u = b^{-1}\big(-f + x_d^{(n)} - k_0\tilde{x} - k_1\dot{\tilde{x}} - \cdots - k_{n-1}\tilde{x}^{(n-1)} - \hat{d}(\tilde{\mathbf{x}})\big) \tag{4}$$

Therefore, the related closed-loop system is:

$$\tilde{x}^{(n)} + k_{n-1}\tilde{x}^{(n-1)} + \ldots + k_1\dot{\tilde{x}} + k_0\tilde{x} = \tilde{d} \tag{5}$$

with $\tilde{d} = \hat{d} - d$.

Due to its simplicity and fast convergence feature, radial basis functions (RBF) are used as activation functions and the related tracking error as input. In this case, the output of the network is defined as:

$$\hat{d}(\tilde{\mathbf{x}}) = \sum_{i=1}^{M} w_i \cdot \varphi_i(||\tilde{\mathbf{x}} - \mathbf{t}||) \tag{6}$$

where $\varphi_i(\cdot)$ are the activation functions and $\mathbf{t}$ a vector containing the coordinates of the center of each activation function.

Now, the signal $\nu = \tilde{x}^{(n)} + k_{n-1}\tilde{x}^{(n-1)} + \ldots + k_1\dot{\tilde{x}} + k_0\tilde{x} - \hat{d}$ is used to train the neural network and the weights of the output layer are adjusted using the pseudo-inverse matrix.

Considering a training set $T = \{(\tilde{\mathbf{x}}, d)_1, (\tilde{\mathbf{x}}, d)_2, \ldots, (\tilde{\mathbf{x}}, d)_p\}$ and

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & \cdots & \varphi_{1M} \\ \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_{p2} & \varphi_{p2} & \cdots & \varphi_{pM} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_p \end{bmatrix} \quad \therefore \quad [\varphi]\{w\} = \{d\} \tag{7}$$

the RBF weights are computed with the pseudo-inverse $[\varphi]^+$

$$\{w\} = [\varphi]^+\{d\} \tag{8}$$

and approximation error, E, by the euclidean norm

$$E = ||\{d\} - [\varphi]\{w\}|| \tag{9}$$

3 Illustrative example: Electro-hydraulic system

Electro-hydraulic actuators play an essential role in several branches of industrial activity and are frequently the most suitable choice for systems that require large forces at high speeds. Their application scope ranges from robotic manipulators to aerospace systems. Another great advantage of hydraulic systems is the ability to keep up the load capacity, which in the case of electric actuators is limited due to excessive heat generation.

However, the dynamic behavior of electro-hydraulic systems is highly nonlinear, which in fact makes the design of controllers for such systems a challenge for the conventional and well established linear control methodologies. In addition to the common nonlinearities that originate from the compressibility of the hydraulic fluid and valve flow-pressure properties, most electro-hydraulic systems are also subjected to hard nonlinearities such as dead-zone due to valve spool overlap.

In order to design the neural network feedback linearization controller, a mathematical model that represents the hydraulic system dynamics is needed. Dynamic models for such systems are well documented in the literature [5].

The electro-hydraulic system considered in this work consists of a four-way proportional valve, a hydraulic cylinder and variable load force. The variable load force is represented by a mass–spring–damper system. The schematic diagram of the system under study is presented in Fig. 1.

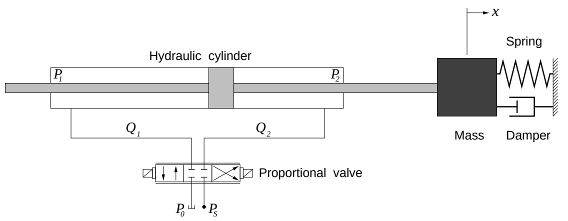


Fig. 1. Schematic diagram of the electro-hydraulic servo-system.

The balance of forces on the piston leads to the following equation of motion:

$$F_g = A_1 P_1 - A_2 P_2 = M_t \ddot{x} + B_t \dot{x} + K_s x \tag{10}$$

where F_g is the force generated by the piston, P_1 and P_2 are the pressures at each side of cylinder chamber, A_1 and A_2 are the ram areas of the two chambers, M_t is the total mass of piston and load referred to piston, B_t is the viscous damping coefficient of piston and load, K_s is the load spring constant and x is the piston displacement.

Defining the pressure drop across the load as $P_L = P_1 - P_2$ and considering that for a symmetrical cylinder $A_p = A_1 = A_2$, Eq. (10) can be rewritten as

$$M_t \ddot{x} + B_t \dot{x} + K_s x = A_p P_L \tag{11}$$

Applying continuity equation to the fluid flow, the following equation is obtained:

$$Q_L = A_p \dot{x} + C_{tp} + \frac{V_t}{4\beta_e} \dot{P}_L \tag{12}$$

where $Q_L = (Q_1 + Q_2)/2$ is the load flow, with Q_1 and Q_2 as the flow in each chamber, C_{tp} the total leakage coefficient of piston, V_t the total volume under compression in both chambers and β_e the effective bulk modulus.

Considering that the return line pressure is usually much smaller than the other pressures involved ($P_0 \approx 0$) and assuming a closed center spool valve with matched and symmetrical orifices, the relationship between load pressure P_L and load flow Q_L can be described as follows

$$Q_L = C_d \omega \bar{x}_{sp} \sqrt{\frac{1}{\rho} \left[P_s - \text{sgn}(\bar{x}_{sp}) P_L \right]} \tag{13}$$

where C_d is the discharge coefficient, ω the valve orifice area gradient, $\bar{x}_{sp}$ the effective spool displacement from neutral, ρ the hydraulic fluid density, P_s the supply pressure and $\text{sgn}(\cdot)$ is defined by

$$\text{sgn}(z) = \begin{cases} -1 \text{ if } & z < 0 \\ 0 \text{ if } & z = 0 \\ 1 \text{ if } & z > 0 \end{cases} \tag{14}$$

Assuming that the dynamics of the valve are fast enough to be neglected, the valve spool displacement can be considered as proportional to the control voltage (u). For closed center valves, or even in the case of the so-called critical valves, the spool presents some overlap. This overlap prevents from leakage losses but leads to a dead-zone nonlinearity within the control voltage, as shown in Fig. 2.

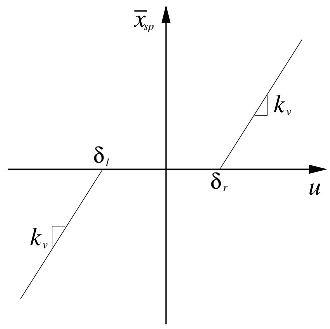


Fig. 2. Dead-zone nonlinearity.

The dead-zone nonlinearity presented in Fig. 2 can be mathematically described by:

$$\bar{x}_{sp}(t) = \begin{cases} k_v\big(u(t)-\delta_l\big) & \text{if} \quad u(t) \le \delta_l \\ 0 & \text{if} \quad \delta_l < u(t) < \delta_r \\ k_v\big(u(t)-\delta_r\big) & \text{if} \quad u(t) \ge \delta_r \end{cases} \tag{15}$$

where k_v is the valve gain and the parameters δ_l and δ_r depends on the size of the overlap region.

For control purposes, as shown by [1], Eq. (15) can be rewritten in a more appropriate form:

$$\bar{x}_{sp}(t) = k_v[u(t) - d] \tag{16}$$

where $d(u)$ can be obtained from Eq. (15) and Eq. (16):

$$d = \begin{cases} \delta_l & \text{if} \quad u(t) \le \delta_l \\ u(t) & \text{if} \quad \delta_l < u(t) < \delta_r \\ \delta_r & \text{if} \quad u(t) \ge \delta_r \end{cases} \tag{17}$$

Combining equations (11), (12), (13), (16) and (17) leads to a third-order differential equation that represents the dynamic behavior of the electro-hydraulic system:

$$\dddot{x} = -\mathbf{a}^{\mathrm{T}}\mathbf{x} + bu - bd \tag{18}$$

where $\mathbf{x} = [x, \dot{x}, \ddot{x}]$ is the state vector with an associated coefficient vector $\mathbf{a} = [a_0, a_1, a_2]$ defined according to

$$a_0 = \frac{4\beta_e C_{tp} K_s}{V_t M_t} \quad ; \quad a_1 = \frac{K_s}{M_t} + \frac{4\beta_e A_p^2}{V_t M_t} + \frac{4\beta_e C_{tp} B_t}{V_t M_t} \quad ; \quad a_2 = \frac{B_t}{M_t} + \frac{4\beta_e C_{tp}}{V_t}$$

and

$$b = \frac{4\beta_e A_p}{V_t M_t} C_d w k_v \sqrt{\frac{1}{\rho}\big[P_s - \mathrm{sgn}(u)\big(M_t\ddot{x} + B_t\dot{x} + K_s x\big)/A_p\big]}$$

In this way, based on Eq. (4), the following nonlinear controller can be proposed to deal with the dynamic model presented in Eq. (18).

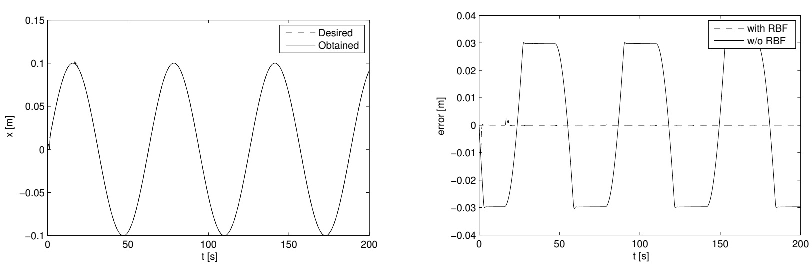
$$u = b^{-1}(\mathbf{a}^{\mathrm{T}}\mathbf{x} + \dddot{x}_d - 3\lambda\ddot{\tilde{x}} - 3\lambda^2\dot{\tilde{x}} - \tilde{x}) + \hat{d}(\tilde{x}, \dot{\tilde{x}}, \ddot{\tilde{x}}) \tag{19}$$

In order to evaluate the control system performance, numerical simulations were carried out. These simulation studies were performed with sampling rates of 500 Hz for control system and 1 kHz for dynamic model. The differential equations of the dynamic model were numerically solved with the fourth order Runge-Kutta method.

The adopted parameters for the electro-hydraulic system were $P_s = 7$ MPa, $\rho = 850$ kg/m^3, $C_d = 0.6$, $\omega = 2.5 \times 10^{-2}$ m, $A_p = 3 \times 10^{-4}$ m^2, $C_{tp} = 2 \times 10^{-12}$ m^3/(s Pa), $\beta_e = 700$ MPa, $V_t = 6 \times 10^{-5}$ m^3, $M_t = 250$ kg, $B_t = 100$ Ns/m, $K_s = 75$ N/m, $\delta_l = -0.5$ V, $\delta_r = 0.5$ V and $\lambda = 8$.

The dead-zone parameters are considered as unknown for the controller design and its effects should be compensated with adopted the neural network.

Regarding the RBF network, the number of neurons in the intermediate layer and the type of the activation functions, as well as how they are distributed over the input space, could be heuristically defined to accommodate designer's experience and experimental knowledge. On this basis, 21 neurons were adopted for the intermediate layer and the activation functions were chosen as of the gaussian type. Figure 3 shows the results obtained with $x_d = 0.5\sin(0.1t)$ m.



(a) Tracking performance. (b) Tracking error.

Fig. 3. Tracking of $x_d = 0.1\sin(0.1t)$ m.

As observed in Fig. 3, despite the unknown dead-zone input, the proposed control scheme allows the electro-hydraulic actuated system to track the desired trajectory with a small tracking error. Through the comparative analysis shown in Fig. 3(b), the improved performance of the proposed controller over the uncompensated counterpart can be easily ascertained.

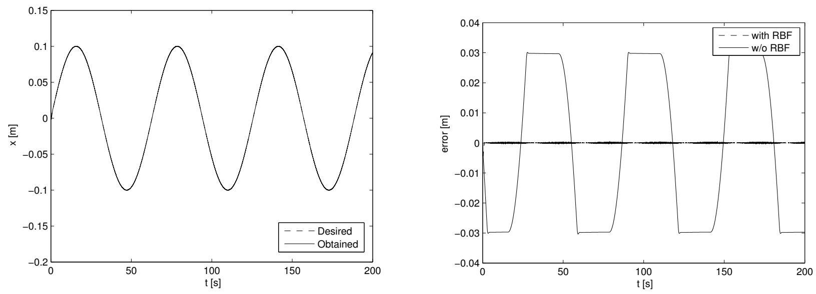
Since noise contamination is unavoidable in experimental data acquisition, it is important to evaluate its effect on control procedures. In order to simulate experimental noisy data sets, a white Gaussian noise was introduced in the signal:

$$x_R(t) = x(t) + \xi$$

$$\dot{x}_R(t) = \dot{x}(t) + \xi$$

$$\ddot{x}_R(t) = \ddot{x}(t) + \xi$$

where x_R, $\dot{x}_R$ and $\ddot{x}_R$ represent the noisy measured state variables, x, $\dot{x}$ and $\ddot{x}$ the related clean signals, and ξ the white Gaussian noise. Here, noise level was parametrized using the signal-to-noise ratio (SNR). Figure 4 show trajectory tracking with SNR = 60.

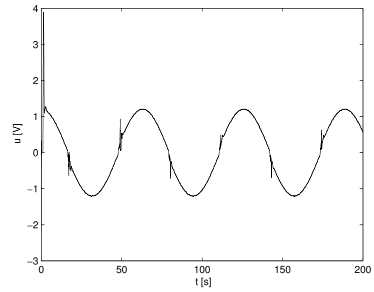


(a) Tracking performance. (b) Tracking error.

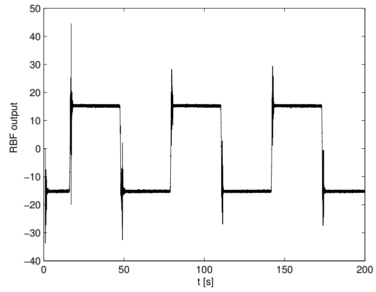
Fig. 4. Tracking of $x_d = 0.1\sin(0.1t)$ m with SNR = 60.

As observed in Fig. 4, even in the presence of noisy measured signals and a dead-zone input the proposed controller is able to provide trajectory tracking. By comparing the tracking error obtained with and without the neural network compensation scheme, Fig. 4(b), the superior performance of the proposed controller is noticeable.

The improved performance of proposed scheme is due to the ability of the RBF network to recognize and compensate for external disturbances and modeling imprecisions. Figure 5 shows the control signal and the output of the network considering SNR = 60.



(a) Control voltage u.



(b) RBF output $\hat{d}$.

Fig. 5. Control signals for the tracking of $x_d = 0.1\sin(0.1t)$ m with SNR = 60.

4 Concluding remarks

The present work addressed the problem of controlling uncertain nonlinear systems with a feedback linearization approach, but enhanced with a RBF neural network. In order to illustrate the controller design method, the proposed scheme is applied to an electro-hydraulic actuated system. The control system performance is confirmed by means of numerical simulations. The adoption of a RBF network provides an smaller tracking error due to its ability to compensate for uncertainties with respect to dynamic model, as well as for noisy signals.

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