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Sliding mode control with adaptive fuzzy dead-zone compensation for uncertain chaotic systems

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Abstract The dead-zone nonlinearity is frequently encountered in many industrial automation equipments and its presence can severely compromise control system performance. In this work, an adaptive variable structure controller is proposed to deal with a class of uncertain nonlinear systems subject to an unknown dead-zone input. The adopted approach is primarily based on the sliding mode control methodology but enhanced by an adaptive fuzzy algorithm to compensate the dead-zone. Using Lyapunov stability theory and Barbalat's lemma, the convergence properties of the closed-loop system are analytically proven. In order to illustrate the controller design methodology, an application of the proposed scheme to a chaotic pendulum is introduced. A comparison between the stabilization of general orbits and unstable periodic orbits embedded in chaotic attractor is carried out showing that the chaos control can confer flexibility to the system by changing the response with low power consumption.

Keywords Chaos control · Dead-zone · Fuzzy logic · Nonlinear pendulum · Sliding modes · Unstable periodic orbits

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1 Introduction

Dead-zone is a hard nonlinearity that can be commonly found in many industrial actuators, especially those containing hydraulic valves and electric motors. Dead-zone characteristics are often unknown and, as previously reported in the research literature, its presence can drastically reduce control system performance and lead to limit cycles in the closed-loop system.

The growing number of papers involving systems with dead-zone input confirms the importance of taking such a non-smooth nonlinearity into account during the control system design process. The most common approaches are adaptive schemes [53, 59, 73, 24, 72, 22, 71, 64, 39, 20, 21], fuzzy systems [26, 43, 30, 7, 38, 74, 54], neural networks [48, 55, 68, 70, 41, 57] and variable structure methods [12, 50]. In many of these works [53, 20, 26, 43, 54, 48, 55], an inverse dead-zone is used to compensate the negative effects of the dead-zone nonlinearity even though this approach leads to a discontinuous control law and requires instantaneous switching, which in practice can not be accomplished with mechanical actuators. To overcome this limitation, smooth inverses are adopted in [73, 72, 71].

An alternative scheme, without using the dead-zone inverse, was originally proposed by Lewis *et al.* [30] and adopted by Wang *et al.* [59] too. In both works, the dead-zone is treated as a combination of a linear and a saturation function. Considering this attractive approach, Bessa *et al.* [7] proposed an adaptive fuzzy compensation scheme to cope with the resulting unknown saturation function.

The dead-zone model presented in [30] was further extended by Ibrir *et al.* [24] and by Zhang and Ge [68], in order to accommodate non-symmetric and unknown dead-zones, respectively. Non-symmetric dead-zones based on [24] are treated in [22,64,39,21,38,74, 57,23] using adaptive methods and the dead-zone model proposed in [68] is also adopted in [70,41,69,49].

The control of chaotic systems subject to a dead-zone input has been likewise investigated [60,42,47,10]. These works demonstrate that variable structure controllers can effectively deal with the deleterious effects of the dead-zone. Moreover, sliding mode control [17,27, 56,66,5] and fuzzy schemes [1,37,46,40,65] have been also applied to chaotic systems.

Sliding mode control is an appealing control technique because of its robustness against both structured and unstructured uncertainties as well as external disturbances. Nevertheless, the discontinuities in the control law must be smoothed out to avoid the undesirable chattering effects. The adoption of properly designed boundary layers have proven effective in completely eliminating chattering, however, leading to an inferior tracking performance. In order to enhance the tracking performance, some adaptive strategy should be used for uncertainty/disturbance compensation.

In this context, considering that fuzzy logic and neural networks can perform universal approximation [13, 19,44,28], much effort has been made to combine these intelligent methodologies with sliding modes [63,52,11, 29,62,67,4] and other nonlinear control schemes [58,61, 18,33,32,35,34,31,36].

On this basis, a generalization of the control scheme presented by Bessa *et al.* [5] is proposed in this paper for a class of n^{th}-order uncertain nonlinear systems subject to unknown dead-zone input. In [5], an adaptive fuzzy inference system is used to approximate the unknown system dynamics within boundary layer of smooth sliding mode controllers. A drawback of this approach, which is also the case in other fuzzy schemes [63,52,11,29,58,18,33,32,35,34], is the adoption of the state variables in the premise of the fuzzy rules. For higher-order systems the number of fuzzy sets and fuzzy rules becomes incredibly large, which compromises the applicability of this technique. In this work, in order to reduce the number of fuzzy sets and rules and consequently simplify the design process, only one variable is considered in the premise of the fuzzy rules. Using Lyapunov's second method and Barbalat's lemma, the convergence properties of the tracking error are analytically proven. Combining the proposed adaptive fuzzy sliding mode controller (AFSMC) with the close return method, the stabilization of unstable periodic orbits (UPOs) is also of concern here. As far as the authors know, the tracking of UPOs in chaotic systems subject to a dead-zone input was not yet examined. As an application of the general procedure, the chaos control of a nonlinear pendulum that has a rich response, presenting chaos and transient chaos [14], is treated. Numerical simulations are carried out illustrating the stabilization of some UPOs of the chaotic attractor showing an effective response. Unstructured uncertainties related to unmodeled dynamics and structured uncertainties associated with parametric variations are both considered in the robustness analysis. Moreover, a comparison between the stabilization of general orbits and unstable periodic orbits embedded in chaotic attractor is performed showing the less energy consumption related to UPOs.

2 Problem statement

Consider a class of n^{th}-order nonlinear systems:

$$x^{(n)} = f(\mathbf{x},t) + h(\mathbf{x},t)\upsilon \tag{1}$$

where the scalar variable $x \in \mathbb{R}$ is the output of interest, $x^{(n)} \in \mathbb{R}$ is the n^{th} derivative of x with respect to time $t \in [0,+\infty)$, $\mathbf{x} = [x, \dot{x}, \ldots, x^{(n-1)}] \in \mathbb{R}^n$ is the system state vector, $f, h : \mathbb{R}^n \to \mathbb{R}$ are both nonlinear functions and $\upsilon \in \mathbb{R}$ represents the output of a dead-zone function $\Upsilon : \mathbb{R} \to \mathbb{R}$, as shown in Fig. 1, with $u \in \mathbb{R}$ stating for the controller output variable.

The adopted dead-zone model is a slightly modified version of that proposed in [68], which can be mathematically described by

$$\upsilon = \Upsilon(u) = \begin{cases} q_l(u) & \text{if} \quad u \leq \delta_l \\ 0 & \text{if} \quad \delta_l < u < \delta_r \\ q_r(u) & \text{if} \quad u \geq \delta_r \end{cases} \tag{2}$$

In respect of the dead-zone model presented in Eq. (2), the following assumptions can be made:

Assumption 1 *The dead-zone output υ is not available to be measured.*

Assumption 2 *The dead-band parameters δ_l and δ_r are unknown but bounded and with known signs, i.e., $\delta_{l\,\min} \leq \delta_l \leq \delta_{l\,\max} < 0$ and $0 < \delta_{r\,\min} \leq \delta_r \leq \delta_{r\,\max}$.*

Assumption 3 *The functions $q_l : (-\infty, \delta_l]$ and $q_r : [\delta_r, +\infty)$ are C^1 and with bounded positive-valued derivatives, i.e.,*

$$0 < p_{l\,\min} \leq q_l'(u) \leq p_{l\,\max}, \quad \forall u \in (-\infty, \delta_l],$$

$$0 < p_{r\,\min} \leq q_r'(u) \leq p_{r\,\max}, \quad \forall u \in [\delta_r, +\infty),$$

where $q_l'(u) = dq_l(z)/dz|_{z=u}$ and $q_r'(u) = dq_r(z)/dz|_{z=u}$.

Remark 1 Assumption 3 means that both q_l and q_r are Lipschitz functions.

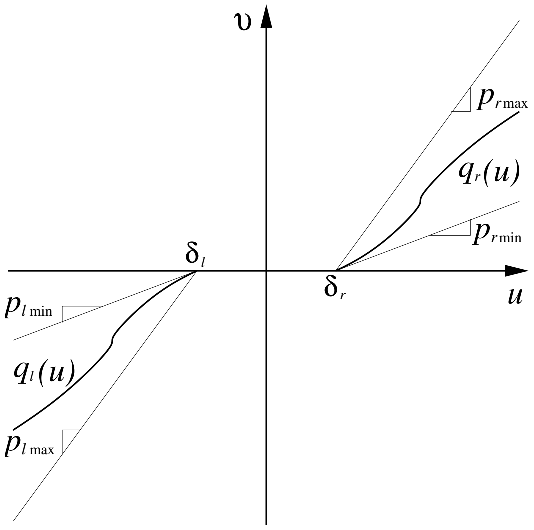


Fig. 1 Dead-zone nonlinearity.

From the mean value theorem and noting that $q_l(\delta_l) = q_r(\delta_r) = 0$, it follows that there exist $\xi_l : \mathbb{R} \to (-\infty, \delta_l)$ and $\xi_r : \mathbb{R} \to (\delta_r, +\infty)$ such that

$$q_l(u) = q_l'\big(\xi_l(u)\big)[u - \delta_l]$$

$$q_r(u) = q_r'\big(\xi_r(u)\big)[u - \delta_r]$$

In this way, Eq. (2) can be rewritten as follows:

$$\upsilon = \Upsilon(u) = \begin{cases} q_l'\big(\xi_l(u)\big)[u - \delta_l] & \text{if} \quad u \leq \delta_l \\ 0 & \text{if} \quad \delta_l < u < \delta_r \\ q_r'\big(\xi_r(u)\big)[u - \delta_r] & \text{if} \quad u \geq \delta_r \end{cases} \tag{3}$$

or in a more appropriate form:

$$\upsilon = \Upsilon(u) = p(u)[u - d(u)] \tag{4}$$

where

$$p(u) = \begin{cases} q_l'\big(\xi_l(u)\big) & \text{if} \quad u \leq 0 \\ q_r'\big(\xi_r(u)\big) & \text{if} \quad u > 0 \end{cases} \tag{5}$$

and

$$d(u) = \begin{cases} \delta_l & \text{if} \quad u \leq \delta_l \\ u & \text{if} \quad \delta_l < u < \delta_r \\ \delta_r & \text{if} \quad u \geq \delta_r \end{cases} \tag{6}$$

Remark 2 Considering Assumption 2 and Eq. (6), it can be easily verified that $d(u)$ is bounded: $|d(u)| \leq \delta$, where $\delta = \max\{-\delta_{l\,\min}, \delta_{r\,\max}\}$.

In respect of the dynamic system presented in Eq. (1), the following assumptions will also be made:

Assumption 4 *The function f is unknown but bounded by a known function of $\mathbf{x}$, i.e., $|\hat{f}(\mathbf{x},t) - f(\mathbf{x},t)| \leq \mathcal{F}(\mathbf{x})$ where $\hat{f}$ is an estimate of f.*

Assumption 5 *The input gain $h(\mathbf{x})$ is unknown but positive and bounded, i.e., $0 < h_{\min} \leq h(\mathbf{x},t) \leq h_{\max}$.*

3 Controller design

As demonstrated in [4], adaptive fuzzy algorithms can be properly embedded in smooth sliding mode controllers to compensate for modeling inaccuracies, in order to improve the trajectory tracking of uncertain nonlinear systems. It has also been shown that adaptive fuzzy sliding mode controllers are suitable for a variety of applications ranging from underwater robotic vehicles [6,8] to electro-hydraulic actuated systems [9].

In this work, the proposed control problem is to ensure that, even in the presence of parametric uncertainties, unmodeled dynamics and an unknown dead-zone input, the state vector $\mathbf{x}$ will follow a desired trajectory $\mathbf{x}_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]$ in the state space.

Regarding the development of the control law, the following assumptions should also be made:

Assumption 6 *The state vector $\mathbf{x}$ is available.*

Assumption 7 *The desired trajectory $\mathbf{x}_d$ is once differentiable in time. Furthermore, every element of vector $\mathbf{x}_d$, as well as $x_d^{(n)}$, is available and with known bounds.*

Now, let $\tilde{x} = x - x_d$ be defined as the tracking error in the variable x, and

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d = [\tilde{x}, \dot{\tilde{x}}, \ldots, \tilde{x}^{(n-1)}]$$

as the tracking error vector.

Consider a sliding surface S defined in the state space by the equation $s(\tilde{\mathbf{x}}) = 0$, with the function $s : \mathbb{R}^n \to \mathbb{R}$ satisfying

$$s(\tilde{\mathbf{x}}) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x}$$

or conveniently rewritten as

$$s(\tilde{\mathbf{x}}) = \mathbf{c}^{\mathrm{T}}\tilde{\mathbf{x}} \tag{7}$$

where $\mathbf{c} = [c_{n-1}\lambda^{n-1}, \dots, c_1\lambda, c_0]$ and c_i states for binomial coefficients, i.e.,

$$c_i = \binom{n-1}{i} = \frac{(n-1)!}{(n-i-1)!\, i!}, \quad i = 0, 1, \dots, n-1 \tag{8}$$

which makes $c_{n-1}\lambda^{n-1} + \cdots + c_1\lambda + c_0$ a Hurwitz polynomial.

From Eq. (8), it can be easily verified that $c_0 = 1$, for $\forall n \geq 1$. Thus, for notational convenience, the time derivative of s will be written in the following form:

$$\dot{s} = \mathbf{c}^{\mathrm{T}}\dot{\tilde{\mathbf{x}}} = \tilde{x}^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}} \tag{9}$$

where $\bar{\mathbf{c}} = [0, c_{n-1}\lambda^{n-1}, \dots, c_1\lambda]$.

Now, let the problem of controlling the uncertain nonlinear system (1) be treated in a Filippov's way [15], defining a control law composed by an equivalent control $\hat{u} = \widehat{hp}^{-1}(-\hat{f} + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})$, an estimate $\hat{d}(\hat{u})$ and a discontinuous term $-K\,\mathrm{sgn}(s)$:

$$u = \widehat{hp}^{-1}(-\hat{f} + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}}) + \hat{d}(\hat{u}) - K\,\mathrm{sgn}(s) \tag{10}$$

where $\widehat{hp} = \sqrt{h_{\max}p_{\max}h_{\min}p_{\min}}$ with $p_{\max} = \max\{p_{l\,\max}, p_{r\,\max}\}$ and $p_{\min} = \min\{p_{l\,\min}, p_{r\,\min}\}$, K is a positive gain and $\mathrm{sgn}(\cdot)$ is defined as

$$\mathrm{sgn}(s) = \begin{cases} -1 \text{ if } & s < 0 \\ 0 \text{ if } & s = 0 \\ 1 \text{ if } & s > 0 \end{cases}$$

Based on Assumptions 2–5 and considering that $\mathcal{H}^{-1} \leq \widehat{hp}/(hp) \leq \mathcal{H}$, where $\mathcal{H} = \sqrt{(h_{\max}p_{\max})/(h_{\min}p_{\min})}$, the gain K should be chosen according to

$$K \geq \mathcal{H}[\widehat{hp}^{-1}(\eta + \mathcal{F}) + \mathcal{H}(\delta + |\hat{d}|) + (\mathcal{H} - 1)|\hat{u}|] \tag{11}$$

where η is a strictly positive constant related to the reaching time.

Therefore, it can be easily verified that (10) is sufficient to impose the sliding condition

$$\frac{1}{2}\frac{d}{dt}s^2 \leq -\eta|s|$$

which, in fact, ensures the finite-time convergence of the tracking error vector to the sliding surface S and, consequently, its exponential stability.

In order to obtain a good approximation to $d(u)$, the estimate $\hat{d}(\hat{u})$ will be computed directly by an adaptive fuzzy algorithm.

The adopted fuzzy inference system was the zero order TSK (Takagi–Sugeno–Kang), whose rules can be stated in a linguistic manner as follows:

If $\hat{u}$ *is* $\hat{U}_i$ *then* $\hat{d}_i = \hat{D}_i$, $\quad i = 1, 2, \dots, N$

where $\hat{U}_i$ are fuzzy sets, whose membership functions could be properly chosen, and $\hat{D}_i$ is the output value of each one of the N fuzzy rules.

At this point, it should be highlighted that the adoption of the equivalent control $\hat{u}$ in the premise of the rules, instead of the state variables as in [5], leads to a smaller number of fuzzy sets and rules, which significantly simplifies the design process. Considering that the dead-zone and external disturbances are independent of the state variables, this choice also seems to be more appropriate in this case.

Considering that each rule defines a numerical value as output $\hat{D}_i$, the final output $\hat{d}$ can be computed by a weighted average:

$$\hat{d}(\hat{u}) = \frac{\sum_{i=1}^{N} w_i \cdot \hat{d}_i}{\sum_{i=1}^{N} w_i} \tag{12}$$

or, similarly,

$$\hat{d}(\hat{u}) = \hat{\mathbf{D}}^{\mathrm{T}}\mathbf{\Psi}(\hat{u}) \tag{13}$$

where, $\hat{\mathbf{D}} = [\hat{D}_1, \hat{D}_2, \dots, \hat{D}_N]$ is the vector containing the attributed values $\hat{D}_i$ to each rule i, $\mathbf{\Psi}(\hat{u}) = [\psi_1(\hat{u}), \psi_2(\hat{u}), \dots, \psi_N(\hat{u})]$ is a vector with components $\psi_i(\hat{u}) = w_i / \sum_{i=1}^{N} w_i$ and w_i is the firing strength of each rule.

In order to ensure the best possible estimate $\hat{d}(\hat{u})$, the vector of adjustable parameters can be automatically updated by the following adaptation law:

$$\dot{\hat{\mathbf{D}}} = -\gamma s \mathbf{\Psi}(\hat{u}) \tag{14}$$

where γ is a strictly positive constant related to the adaptation rate.

It is important to emphasize that the chosen adaptation law, Eq. (14), must not only provide a good approximation to $d(u)$ but also not compromise the attractiveness of the sliding surface, as will be proven in the following theorem.

Theorem 1 *Consider the uncertain nonlinear system (1) subject to the dead-zone (4) and Assumptions 1–7. Then, the controller defined by (10), (11), (13) and (14) ensures the convergence of the tracking error vector to the sliding surface S.*

Proof Let a positive-definite function V be defined as

$$V(t) = \frac{1}{2}s^2 + \frac{\mathcal{H}\widehat{hp}}{2\gamma}\mathbf{\Delta}^{\mathrm{T}}\mathbf{\Delta}$$

where $\mathbf{\Delta} = \hat{\mathbf{D}} - \hat{\mathbf{D}}^*$ and $\hat{\mathbf{D}}^*$ is the optimal parameter vector, associated to the optimal estimate $\hat{d}^*(\hat{u})$. Thus, the time derivative of V is

$$\begin{aligned}\dot{V}(t) &= s\dot{s} + \mathcal{H}\widehat{hp}\gamma^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}}\\ &= (\tilde{x}^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})s + \mathcal{H}\widehat{hp}\gamma^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}}\\ &= (x^{(n)} - x_d^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})s + \mathcal{H}\widehat{hp}\gamma^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}}\\ &= (f + hpu - hpd - x_d^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})s + \mathcal{H}\widehat{hp}\gamma^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}}\\ &= [f + hp\widehat{hp}^{-1}(-\hat{f} + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}}) + hp\hat{d}\\ &\quad - hpK\,\mathrm{sgn}(s) - hpd - (x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})]s +\\ &\quad + \mathcal{H}\widehat{hp}\gamma^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}}\end{aligned}$$

Defining the minimum approximation error as $\varepsilon = \hat{d}^* - d$, recalling that $\hat{u} = \widehat{hp}^{-1}(-\hat{f} + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})$, and noting that $\dot{\mathbf{\Delta}} = \dot{\hat{\mathbf{D}}}$ and $f = \hat{f} - (\hat{f} - f)$, $\dot{V}$ becomes:

$$\begin{aligned}\dot{V}(t) &= -[(\hat{f} - f) - hp\varepsilon - hp(\hat{d} - \hat{d}^*) + \widehat{hp}\hat{u} - hp\hat{u}+\\ &\quad + hpK\,\mathrm{sgn}(s)]s + \mathcal{H}\widehat{hp}\gamma^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\hat{\mathbf{D}}}\\ &= -[(\hat{f} - f) - hp\varepsilon - hp(\hat{\mathbf{D}} - \hat{\mathbf{D}}^*)\mathbf{\Psi}(\hat{u}) + \widehat{hp}\hat{u}+\\ &\quad - hp\hat{u} + hpK\,\mathrm{sgn}(s)]s + \mathcal{H}\widehat{hp}\gamma^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\hat{\mathbf{D}}}\\ &= -[(\hat{f} - f) - hp\varepsilon - hp\mathbf{\Delta}^{\mathrm{T}}\mathbf{\Psi}(\hat{u}) + \widehat{hp}\hat{u} - hp\hat{u}+\\ &\quad + hpK\,\mathrm{sgn}(s)]s + \mathcal{H}\widehat{hp}\gamma^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\hat{\mathbf{D}}}\end{aligned}$$

Since $\mathcal{H}^{-1} \leq \widehat{hp}/(hp) \leq \mathcal{H}$, it follows that

$$\begin{aligned}\dot{V}(t) &\leq -[(\hat{f} - f) - \mathcal{H}\widehat{hp}\varepsilon + \widehat{hp}\hat{u} - \mathcal{H}\widehat{hp}\hat{u}+\\ &\quad + \mathcal{H}^{-1}\widehat{hp}K\,\mathrm{sgn}(s)]s + \mathcal{H}\widehat{hp}\gamma^{-1}\mathbf{\Delta}^{\mathrm{T}}[\dot{\hat{\mathbf{D}}} + \gamma s\mathbf{\Psi}(\hat{u})]\end{aligned}$$

Thus, by applying the adaptation law (14) to $\dot{\hat{\mathbf{D}}}$:

$$\dot{V}(t) \leq -[(\hat{f}-f) - \mathcal{H}\widehat{hp}\varepsilon + \widehat{hp}\hat{u} - \mathcal{H}\widehat{hp}\hat{u} + \mathcal{H}^{-1}\widehat{hp}K\,\mathrm{sgn}(s)]s$$

Furthermore, considering Assumptions 2–5, defining K according to (11) and verifying that $|\varepsilon| = |\hat{d}^* - d| \leq |\hat{d} - d| \leq |\hat{d}| + \delta$, one has:

$$\dot{V}(t) \leq -\eta|s| \tag{15}$$

which implies $V(t) \leq V(0)$ and that s and $\mathbf{\Delta}$ are bounded. Considering that $s(\tilde{\mathbf{x}}) = \mathbf{c}^{\mathrm{T}}\tilde{\mathbf{x}}$, it can be verified that $\tilde{\mathbf{x}}$ is also bounded. Hence, equation (9) and Assumption 7 implies that $\dot{s}$ is also bounded.

Integrating both sides of (15) shows that

$$\lim_{t\to\infty}\int_0^t \eta|s|\,d\tau \leq \lim_{t\to\infty}[V(0) - V(t)] \leq V(0) < \infty$$

Since the absolute value function is uniformly continuous, it follows from Barbalat's lemma [25] that $s \to 0$ as $t \to \infty$, which ensures the convergence of the tracking error vector to the sliding surface S and completes the proof.

However, the presence of a discontinuous term in the control law leads to the well known chattering phenomenon. To overcome the undesirable chattering effects, Slotine [51] proposed the adoption of a thin boundary layer, S_ϕ, in the neighborhood of the switching surface:

$$S_\phi = \left\{\tilde{\mathbf{x}} \in \mathbb{R}^n \;\middle|\; |s(\tilde{\mathbf{x}})| \leq \phi\right\} \tag{16}$$

where ϕ is a strictly positive constant that represents the boundary layer thickness.

The boundary layer is achieved by replacing the sign function by a continuous interpolation inside S_ϕ. It should be noted that this smooth approximation, which will be called here $\varphi(s,\phi)$, must behave exactly like the sign function outside the boundary layer. There are several options to smooth out the ideal relay but the most common choices are the saturation function:

$$\mathrm{sat}(s/\phi) = \begin{cases}\mathrm{sgn}(s) & \text{if} \quad |s/\phi| \geq 1\\ s/\phi & \text{if} \quad |s/\phi| < 1\end{cases} \tag{17}$$

and the hyperbolic tangent function $\tanh(s/\phi)$.

In this way, to avoid chattering, a smooth version of Eq. (10) can be adopted:

$$u = \widehat{hp}^{-1}(-\hat{f} + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}}) + \hat{d}(\hat{u}) - K\varphi(s,\phi) \tag{18}$$

Nevertheless, it should be emphasized that the substitution of the discontinuous term by a smooth approximation inside the boundary layer turns the perfect tracking into a tracking with guaranteed precision problem, which actually means that a steady-state error will always remain. According to [3], the tracking error vector will exponentially converge to a closed region $\Phi = \{\tilde{\mathbf{x}} \in \mathbb{R}^n \mid |s(\tilde{\mathbf{x}})| \leq \phi \text{ and } |\tilde{x}^{(i)}| \leq \sigma_i \lambda^{i-n+1}\phi, i = 0, 1, \ldots, n-1\}$, with σ_i defined as

$$\sigma_i = \begin{cases} 1 & \text{for} \quad i = 0 \\ 1 + \sum_{j=0}^{i-1} \binom{i}{j} \sigma_j & \text{for} \quad i = 1, 2, \ldots, n-1. \end{cases} \tag{19}$$

Since all sliding mode parameters (λ, η and ϕ) have either geometric or physical interpretation, the values for these parameters can be adjusted in an easy and straightforward manner. Moreover, the adoption of the equivalent control $\hat{u}$ in the premise of the rules, instead of the state variables, lead to a smaller number of fuzzy sets and rules, which in fact simplify the control system design process. In this way, it is important to stress that these features facilitate the implementation of the proposed adaptive fuzzy sliding mode controller in real world applications.

4 Chaotic pendulum

As an application of the control procedure, a nonlinear pendulum is investigated. This pendulum is based on an experimental set up, previously analyzed by Franca and Savi [16] and Pereira-Pinto *et al.* [45]. De Paula *et al.* [14] presented a mathematical model to describe the dynamical behavior of the pendulum and the corresponding experimentally obtained parameters.

The schematic picture of the considered nonlinear pendulum is shown in Fig. 2. Basically, the pendulum consists of an aluminum disc (1) with a lumped mass (2) that is connected to a rotary motion sensor (4). This assembly is driven by a string-spring device (6) that is attached to an electric motor (7) and also provides torsional stiffness to the system. A magnetic device (3) provides an adjustable dissipation of energy. An actuator (5) provides the necessary perturbations to stabilize this system by properly changing the string length.

In order to obtain the equations of motion of the experimental nonlinear pendulum it is assumed that system dissipation may be expressed by a combination of a linear viscous dissipation together with dry friction. Therefore, denoting the angular position as θ, the following equation is obtained [14]:

$$\begin{aligned} &I\ddot{\theta} + \zeta\dot{\theta} + \mu\,\mathrm{sgn}(\dot{\theta}) + 2kr^2\theta + mgR\sin(\theta) = \\ &= kr\left[\sqrt{a^2 + b^2 - 2ab\cos(\omega t)} - (a-b) - \Delta l\right] \end{aligned} \tag{20}$$

where ω is the forcing frequency related to the motor rotation, a defines the position of the guide of the string with respect to the motor, b is the length of the excitation crank of the motor, R is the radius of the metallic disc and r is the radius of the driving pulley, m is the lumped mass, ζ represents the linear viscous damping coefficient, while μ is the dry friction coefficient; g is the gravity acceleration, I is the inertia of the disk-lumped mass, k is the string stiffness and Δl is the length variation in the spring provided by the linear actuator (5).

De Paula *et al.* [14] show that this mathematical model presents results that are in close agreement with experimental data. The pendulum equation can be expressed in terms of Eq. (1) by assuming that $\mathbf{x} = [\theta, \dot{\theta}]$, $h = kr/I$ and $v = -\Delta l$. The function f can be obtained from Eq. (1) and Eq. (20).

5 Controlling the chaotic pendulum

In order to illustrate the controller design method and to demonstrate its performance, consider the chaotic pendulum, mathematically described by Eq.(20), with a dead-zone input defined by

$$\upsilon = \begin{cases} 0.9(u + 0.003) + 0.001\sin(u) & \text{if} \quad u \leq \delta_l \\ 0 & \text{if} \quad \delta_l < u < \delta_r \\ 1.0(u - 0.002) - 0.001\cos(u) & \text{if} \quad u \geq \delta_r \end{cases} \tag{21}$$

where $\delta_l = -0.003$ and $\delta_r = 0.002$.

On this basis, according to the previously described control scheme and considering $s = \dot{\tilde{\theta}} + \lambda\tilde{\theta}$, with $\tilde{\theta} = \theta - \theta_d$ as the tracking error, $\dot{\tilde{\theta}}$ as the first time derivative of $\tilde{\theta}$ and θ_d as the desired trajectory, a smooth control law can be defined as follows

$$u = \widehat{hp}^{-1}(-\hat{f} + \ddot{\theta}_d - \lambda\dot{\tilde{\theta}}) + \hat{d}(\hat{u}) - K\,\mathrm{sat}(s/\phi)$$

The controller capability is now investigated by considering numerical simulations. The fourth order Runge-Kutta method is employed and sampling rates of 107 Hz for control system and 214 Hz for dynamical model are assumed. The model parameters are chosen according to [14]: $I = 1.738 \times 10^{-4}$ kg m^2; $m = 1.47 \times 10^{-2}$ kg; $k = 2.47$ N/m; $\zeta = 2.368 \times 10^{-5}$ kg m^2/s; $\mu = 1.272 \times 10^{-4}$ N m; $a = 1.6 \times 10^{-1}$ m; $b = 6.0 \times 10^{-2}$ m; $r = 2.4 \times 10^{-2}$ m; $R = 4.75 \times 10^{-2}$ m and $\omega = 5.61$ rad/s. Regarding controller parameters, the following values were chosen: $\mathcal{F} = 1.2$; $\mathcal{H} = 1.1$; $\delta = 0.003$, $\phi = 1.0$; $\lambda = 0.8$; $\eta = 0.05$ and $\gamma = 0.4$.

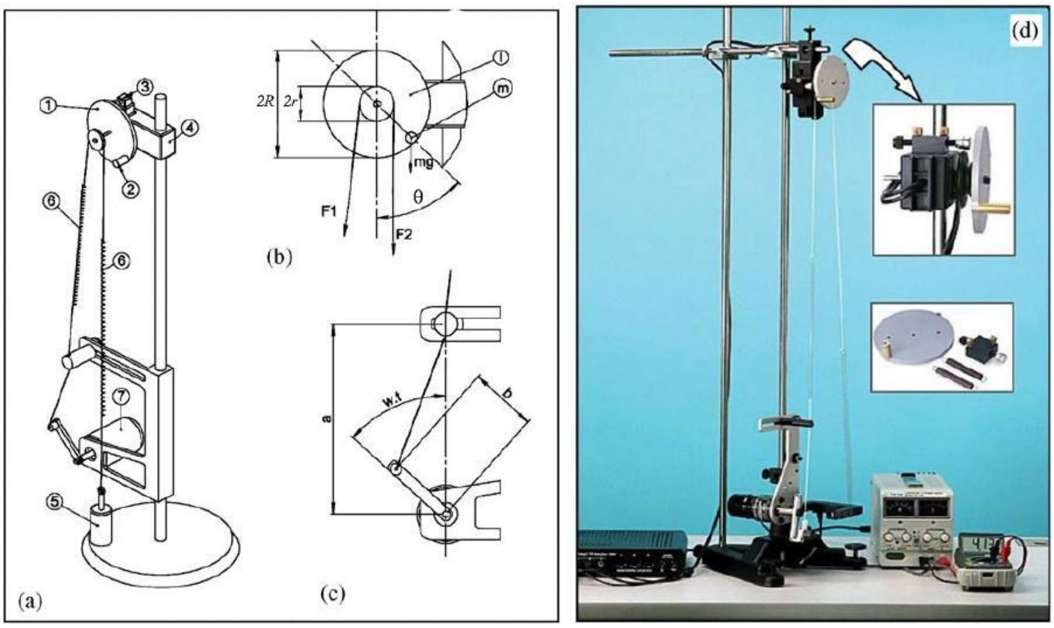


Fig. 2 (a) Nonlinear pendulum – (1) metallic disc; (2) lumped mass; (3) magnetic damping device; (4) rotary motion sensor (PASCO CI-6538); (5) anchor mass; (6) string-spring device; (7) electric motor (PASCO ME-8750). (b) Parameters and forces on metallic disc. (c) Parameters from driving device. (d) Experimental apparatus.

Concerning the fuzzy inference system, the number of fuzzy rules and the type of the membership functions, as well as how they are distributed over the input space, could be heuristically defined to accommodate designer's experience and experimental knowledge. On this basis, assuming no previous knowledge about d, seven rules (with seven related fuzzy sets $\hat{U}_r$) were arbitrarily chosen and placed within the input space $\hat{u}$. Triangular and trapezoidal membership functions were adopted for $\hat{U}_r$, with the central values defined as $C = \{-1.0; -0.5; -0.1; 0.0; 0.1; 0.5; 1.0\} \times 10^{-2}$ (see Fig. 3). It should be stressed that the input space could be partitioned and represented in many other ways, and that the system designer may test each one of them in order to improve the output value $\hat{d}(\hat{u})$. Concerning the vector of adjustable parameters, it was initialized with zero values, $\hat{\mathbf{D}} = \mathbf{0}$, and updated at each iteration step according to the adaptation law presented in Eq. (14).

For tracking purposes, different UPOs are identified using the close return method [2] and three of these are chosen as desired trajectories in the numerical studies that follows.

First, in order demonstrate that the adopted control scheme can deal with unknown dead-zones, it is assumed that model parameters are perfectly known but the dead-zone, Eq. (21), is not taken into account within the design of the control law. Figures 4 and 5 give the corresponding results for the stabilization of two unstable periodic orbits.

As observed in Fig. 4 and Fig. 5, even in the presence of an unknown dead-zone input, the adaptive fuzzy sliding mode controller (AFSMC) is capable to provide the trajectory tracking with a small associated error. It should be emphasized that the control action u represents the length variation in the string and only tiny variations are required to provide such different dynamic behaviors, which actually allows a great flexibility for the controlled nonlinear system.

It can be also verified that the proposed control law provides a smaller tracking error when compared with the conventional sliding mode controller (SMC), Fig. 4(d) and Fig. 5(d). By considering simulation purposes, the AFSMC can be easily converted to the classical SMC by setting the adaptation rate to zero, $\gamma = 0$.

At this point, it is assumed that the dry friction is completely unknown and the viscous damping coefficient is not exactly known. The idea is to ratify the robustness of the adopted control scheme against both unstructured uncertainties (or unmodeled dynamics) and structured (or parametric) uncertainties. On this basis, the estimate $\hat{\mu}$ is taken to vanish, $\hat{\mu} = 0$, and an uncertainty of $\pm 20\%$ over the viscous damping coefficient, $\hat{\zeta} = 1.9 \times 10^{-5}$ kg m^2/s is chosen for the computation of $\hat{f}$ in the control law. The other controller parameters are chosen as before. Therefore, four different situations are treated. In the first case, Fig. 6(a) and Fig. 6(b), a general artificial orbit $[\theta_d, \dot{\theta}_d] = [1.0 + 2.35\sin(2\pi t), 4.70\pi\cos(2\pi t)]$ is considered. A second case, on the other hand, stabilizes a period-1 UPO, Fig. 6(c)

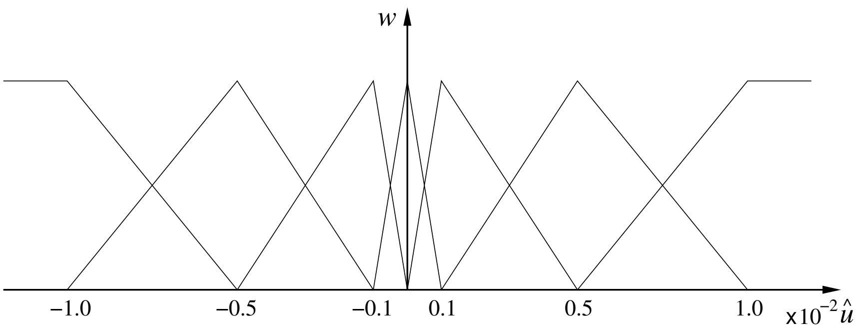


Fig. 3 Adopted fuzzy membership functions.

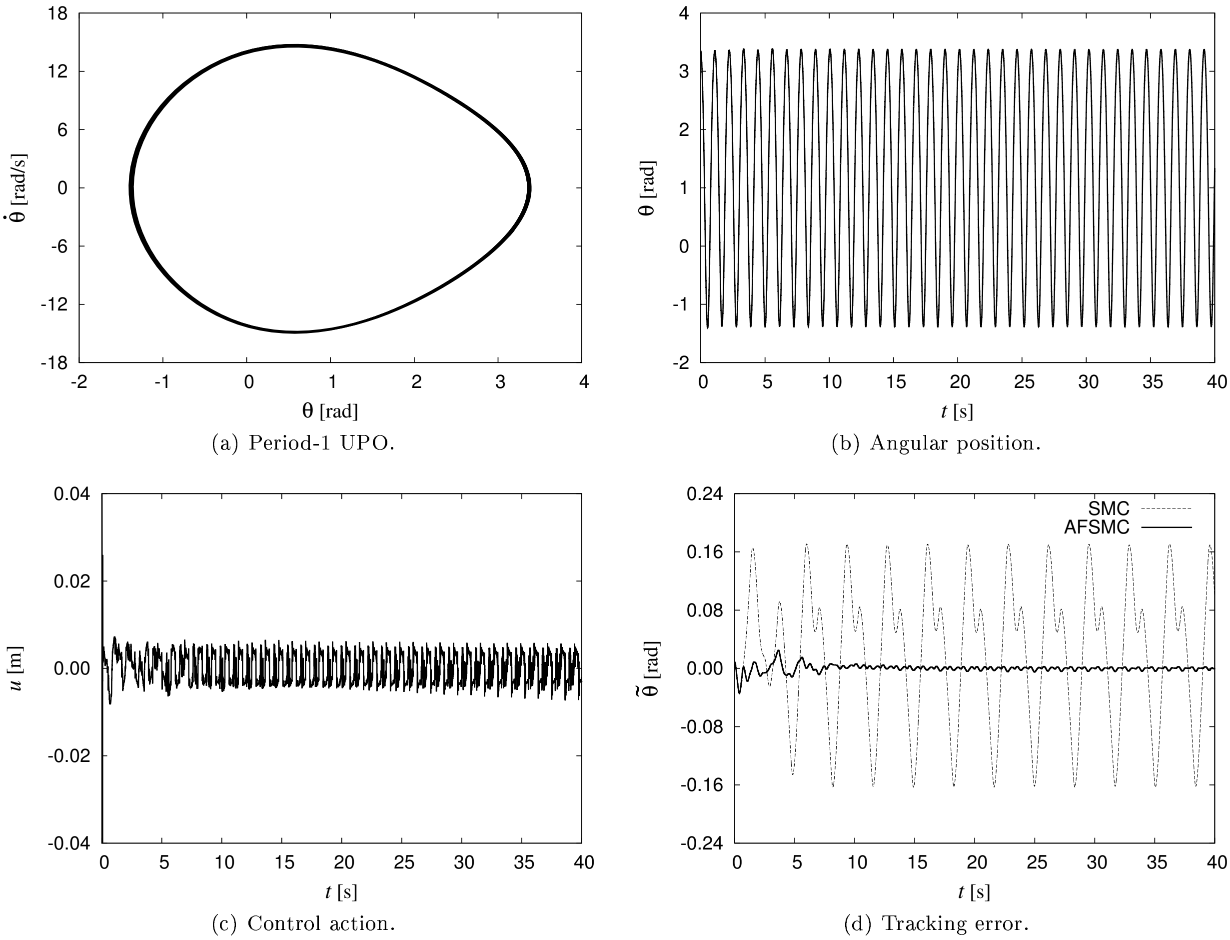
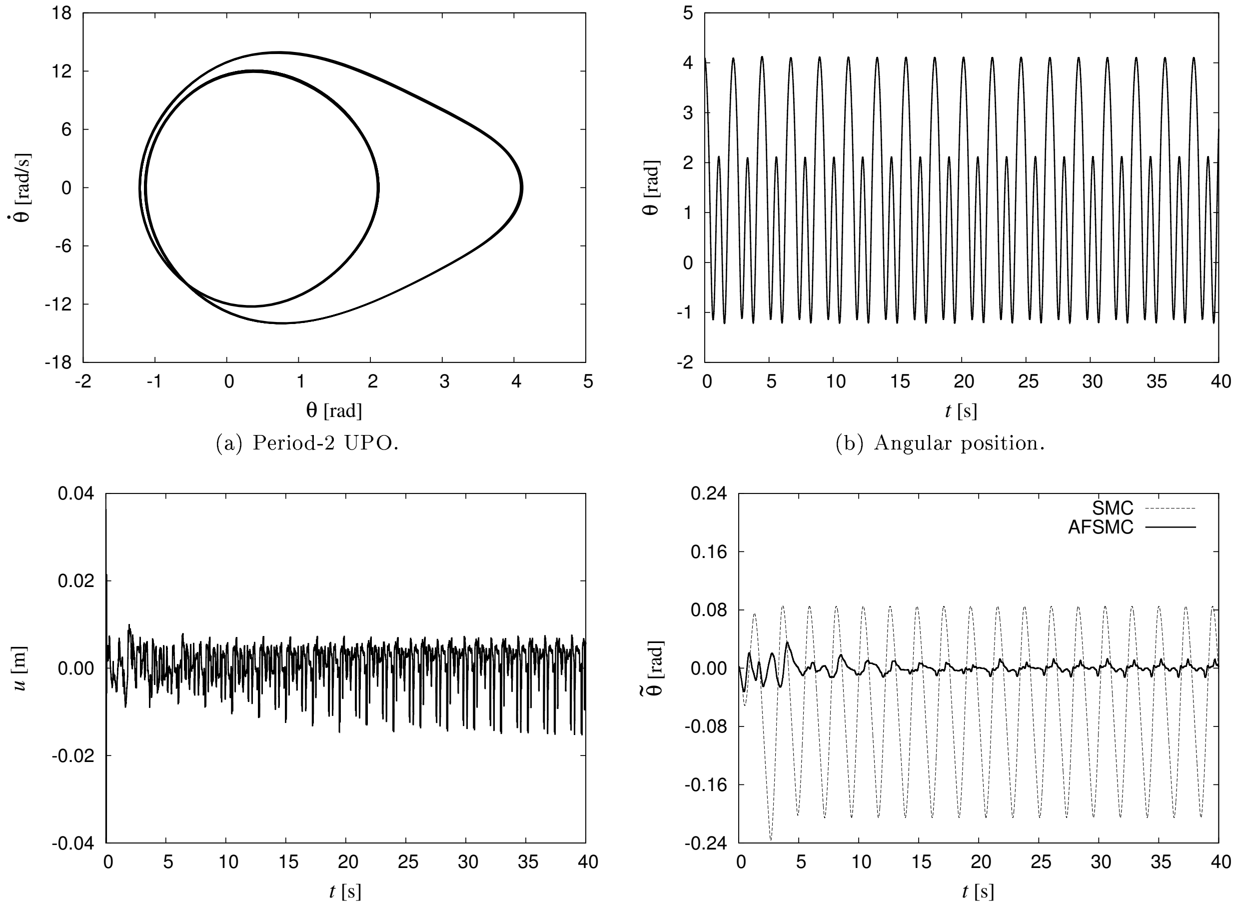


Fig. 4 Tracking of a period-1 UPO.

and Fig. 6(d). Although both orbits are similar, it should be highlighted that the controller requires less effort to stabilize the UPO. Even with more complicated orbits, as is the case of the period-2 UPO shown in Fig. 6(e) and the period-4 UPO shown in Fig. 6(g), the amplitudes of the control actions, Fig. 6(f) and Fig. 6(h), respectively, are significantly smaller when compared with the control effort required to stabilize the general orbit. The control of unstable periodic orbits is the essential aspect to be explored in chaos control that can confer flexibility to the system with low energy consumption.

6 Concluding remarks

The present work addresses the problem of controlling uncertain nonlinear systems subject to an unknown dead-zone input. An adaptive fuzzy sliding mode controller is proposed to deal with the stabilization of un-



(a) Period-2 UPO.

(b) Angular position.

(c) Control action.

(d) Tracking error.

Fig. 5 Tracking of a period-2 UPO.

stable periodic orbits of chaotic systems. The adoption of the equivalent control $\hat{u}$ in the premise of the rules, instead of the state variables, led to a smaller number of fuzzy sets and rules. The convergence properties of the closed-loop system are analytically proven using Lyapunov stability theory and Barbalat's lemma. To illustrate the controller design method and to evaluate its performance, the AFSMC is applied to a chaotic pendulum. By means of numerical simulations, it could be verified that the proposed scheme is able to cope with the dead-zone effects, as well as with both structured and unstructured uncertainties. The improved performance over the conventional sliding mode controller is demonstrated. This work also confirms that less effort is needed to stabilize an UPO when compared with a general non-natural orbit.

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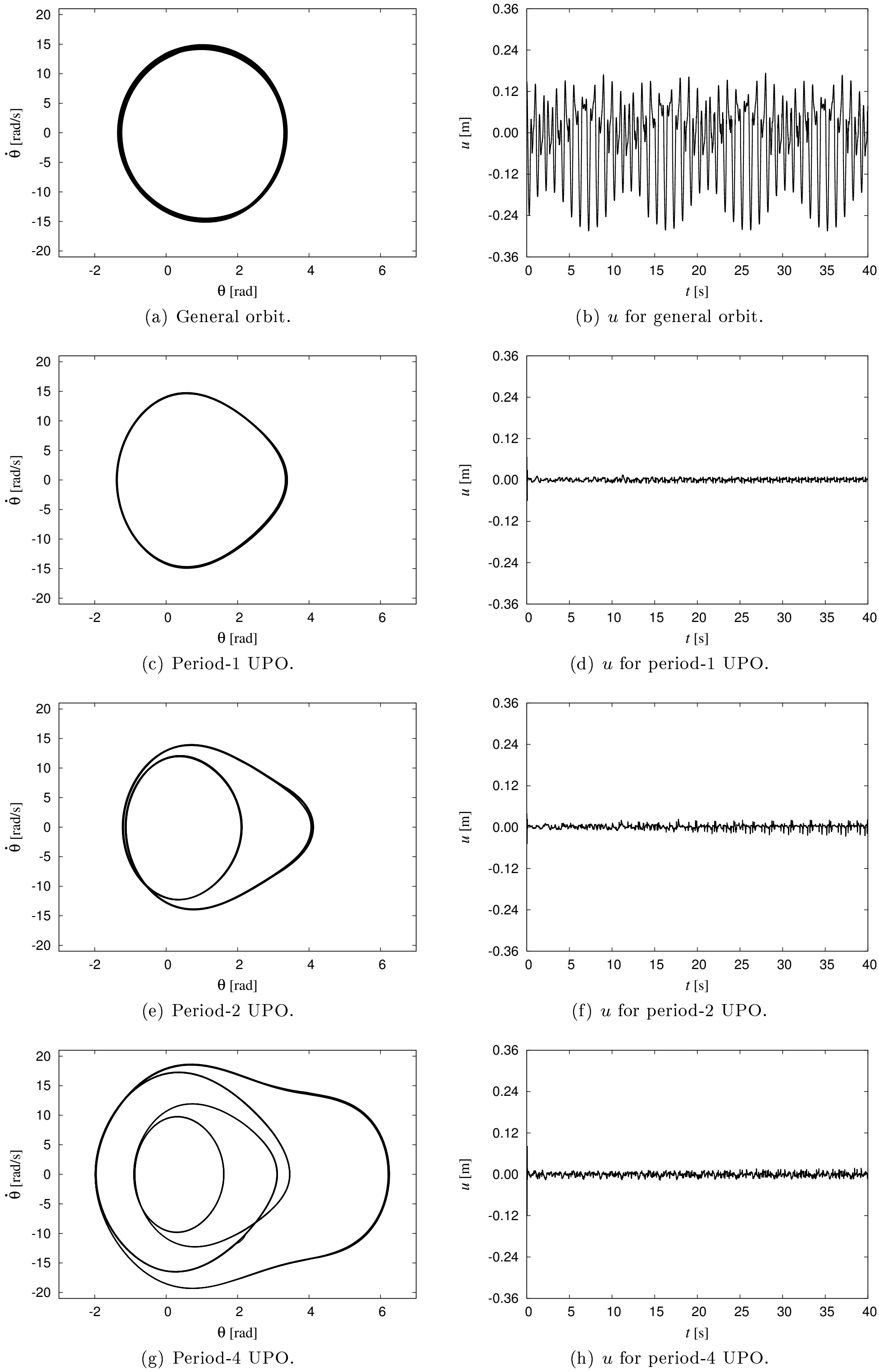

(a) General orbit. (b) u for general orbit.

(c) Period-1 UPO. (d) u for period-1 UPO.

(e) Period-2 UPO. (f) u for period-2 UPO.

(g) Period-4 UPO. (h) u for period-4 UPO.

Fig. 6 Stabilization of a general orbit and 3 different UPOs.

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