# Adaptive fuzzy sliding mode control of the cart-pole underactuated system

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In this work, an intelligent control scheme is proposed for the stabilization of the cart-pole underactuated system. The adopted approach is primarily based on a smooth sliding mode controller, but an adaptive fuzzy inference system is embedded within the boundary layer in order to improve the control performance.



## 1 Introduction

A mechanical system could be defined as underactuated if it has more degrees of freedom to be controlled than independent control inputs/actuators. Underactuated mechanical systems (UMS) play an essential role in several branches of industrial activity and their application scope ranges from robotic manipulators and overhead cranes to aerospace vehicles and watercrafts.

Despite the broad spectrum of applications, the problem of designing accurate controllers for underactuated systems is, however, much more tricky than for fully actuated ones. Moreover, the dynamic behavior of an UMS is frequently uncertain and highly nonlinear, which in fact makes the design of control schemes for such systems a challenge for conventional and well established methods. In this context, a sliding mode based approach is proposed in [1] for the stabilization of underactuated mechanical systems. Sliding mode control (SMC) is an appealing technique because of its robustness against modeling inaccuracies. Nevertheless, the discontinuities in the control law must be smoothed out to avoid the undesirable chattering effects. The adoption of properly designed boundary layers have proven to be effective in completely eliminating chattering, however, leading to an inferior control performance.

In this work, an adaptive fuzzy sliding mode controller (AFSMC) is proposed for a cart-pole underactuated system with unmodeled dynamics. The adopted approach is based on the control strategy defined in [1], but an adaptive fuzzy inference system is embedded within the boundary layer to improve balancing efficiency.

## 2 Stabilizing the cart-pole underactuated system

The cart-pole system is composed by a small car with an inverted pendulum on it (see Fig. 1), and the related equations of motion are presented as

$$\begin{bmatrix} m_c + m & ml\cos\theta \\ ml\cos\theta & ml^2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} ml\dot{\theta}^2\sin\theta \\ mgl\sin\theta \end{bmatrix} + \begin{bmatrix} u \\ 0 \end{bmatrix}. \tag{1}$$

Here, $x$ and $\theta$ are, respectively, the position of the cart and the angular displacement of the pendulum, $m_c$ is the mass of the cart, $m$ and $l$ represent the concentrated mass and the length of the pendulum, and $g$ is the acceleration due to gravity.

It should be highlighted that, since only the position of the cart $x$ can be directly controlled, the angular displacement $\theta$ is considered an unactuated variable. On this basis, Ashrafiuon and Erwin [1] defined the switching variable as a linear combination of both actuated and unactuated state errors, $s = \alpha_a\dot{\tilde{x}} + \lambda_a\tilde{x} + \alpha_u l\dot{\tilde{\theta}} + \lambda_u l\tilde{\theta}$, with $\tilde{x} = x - x_d$ and $\tilde{\theta} = \theta - \theta_d$ as well as their derivatives representing the state errors. But considering that large uncertainties and external disturbances may occur, we suggest the inclusion of a compensation term $\hat{d}$ in the control law proposed in [1], in order to compensate for modeling inaccuracies and to improve the control performance:

$$u = -\frac{(m_c + m\sin^2\theta)l}{\alpha_a l - \alpha_u\cos\theta}\left[\frac{(\alpha_a l - \alpha_u\cos\theta)ml\dot{\theta}^2\sin\theta - [\alpha_a lm\cos\theta - \alpha_u(mc+m)]g\sin\theta}{(m_c + m\sin^2\theta)l} + \dot{s}_r + \hat{d} + K\,\mathrm{sat}\left(\frac{s}{\phi}\right)\right] \tag{2}$$

where $K$ is the control gain, $\phi$ defines the width of the boundary layer, and $\dot{s}_r = -\alpha_a\ddot{x}_d - \alpha_u l\ddot{\theta}_d + \lambda_a\dot{\tilde{x}} + \lambda_u l\dot{\tilde{\theta}}$.

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The compensation term is computed by an adaptive fuzzy algorithm: $\hat{d}(s) = \hat{\mathbf{D}}^{\mathrm{T}}\mathbf{\Psi}(s)$, where $\hat{\mathbf{D}} = [\hat{D}_1, \hat{D}_2, \ldots, \hat{D}_N]^{\mathrm{T}}$ is the vector containing the attributed values $\hat{D}_r$ to each rule $r$, $\mathbf{\Psi}(s) = [\psi_1(s), \psi_2(s), \ldots, \psi_N(s)]^{\mathrm{T}}$ is a vector with components $\psi_r(s) = w_r / \sum_{r=1}^{N} w_r$ and $w_r$ is the firing strength of each rule. The vector of adjustable parameters is automatically updated by the adaptation law $\dot{\hat{\mathbf{D}}} = \varphi s \mathbf{\Psi}(s)$, where $\varphi$ is a strictly positive constant related to the adaptation rate [2].

The performance of the proposed scheme is now investigated by means of numerical simulations. Concerning the fuzzy system, triangular (in the middle) and trapezoidal (at the edges) membership functions are adopted, with the central values defined as $C = \{-\phi/4, -\phi/20, -\phi/40, 0, \phi/40, \phi/20, \phi/4\}$, $\phi = 0.05$ and $\varphi = 200$. The other controller and model parameters are chosen as in [1]. Figures 2 and 3 show the obtained results.

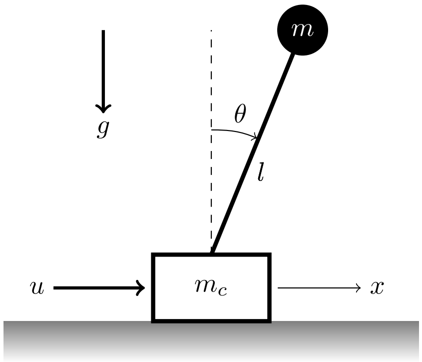
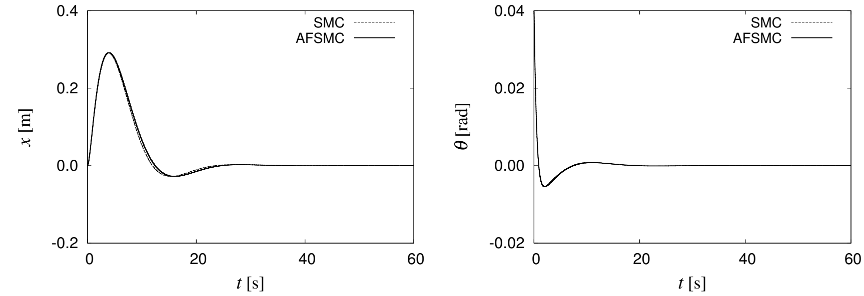
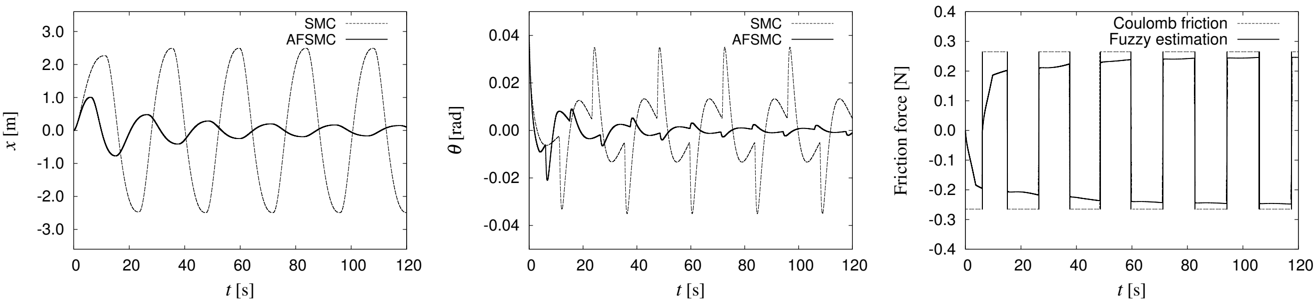



**Fig. 1:** Cart-pole system.




**(a)** Cart position. **(b)** Pendulum angle.

**Fig. 2:** Stabilizing the cart-pole with only parametric uncertainties.




**(a)** Cart position. **(b)** Pendulum angle. **(c)** Fuzzy estimation.

**Fig. 3:** Stabilizing the cart-pole with parametric uncertainties and unmodeled dynamics.

As observed in Fig. 2, if only parametric uncertainties are considered (15% over the cart and pendulum masses), both control schemes (with and without adaptive fuzzy compensation) show a closely similar performance. On the other hand, as demonstrated in Fig 3, if not only parametric uncertainties but also unmodeled dynamics is taken into account, the improved balancing efficiency of the adaptive fuzzy sliding mode controller can be clearly ascertained. In this case, Coulomb friction is treated as unmodeled dynamics. The improved performance of the proposed scheme is due to its ability to recognize and compensate for modelling inaccuracies (Fig. 3c).

## 3 Concluding remarks

The stabilization of the cart-pole underactuated system is handled in this work by an adaptive fuzzy sliding mode controller. The adopted scheme shows the ability to learn from its own experience in order to improve balancing performance when plant model is subject to large uncertainties.

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## References

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